Market Segmentation and Non-Parametric Asset Pricing

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Abstract

I propose to examine market segmentation between equity and bond markets. Although under the no-arbitrage principle equity and bond markets should prove integrated, real-world frictions may induce some degree of market segmentation. To assess the extent of this segmentation, I will use non-parametric estimators of the stochastic discount factor (SDF). I propose to make four contributions in this work. First, the non-parametric methods I will use surmount several econometric limitations of previous such investigations. Second, I propose a novel machine learning-based SDF estimator. Third, I intend to examine time variation in the extent of segmentation between equity and bond markets, which previous work has not empirically tested. Fourth, I will use dual-asset-class SDF estimates to examine cross-asset-class trading signals, which have immediate practical applications.
1 Introduction

Asset prices equal expected discounted future payoffs. All of asset pricing stems from this principle. In theory, under the no-arbitrage condition, there exists a stochastic discount factor (SDF) - a strictly positive random variable that discounts random future payoffs - that prices all assets (Hansen & Richard, 1987). Such an SDF, if correctly estimated, should explain the variation in expected returns across any cross-section of assets. Thus, in theory, a single SDF should price the cross-section of equities and bonds. Of course, in theory there is no difference between theory and reality. In reality, on the other hand, it is possible that equity and bond markets are “segmented.” Market segmentation arises when investors do not necessarily trade across asset classes, and can lead to different pricing properties. I seek to investigate the extent of market segmentation between equity and bond markets by answering the following question: Do bond yields contain information that can help explain the cross-section of equity returns and vice versa? Specifically, I will use non-parametric estimates of the SDF to determine if an SDF estimated from equities and bonds possesses greater explanatory power than one derived from a single asset class.

My examination of this question interacts with three veins of the existing asset pricing literature. First is previous research on market segmentation in a variety of settings. Most prior work in market segmentation has focused on segmentation in international equity markets and segmentation across the term structure. Second is research on cross-sectional stock and bond pricing. Stocks command a risk premium because they often perform poorly in times of high marginal utility for investors, such as recessions or other negative economic events (Cochrane, 2017). Thus, to the extent bond yields capture investor expectations of future economic activity, they should serve as proxies for priced risk factors in the cross-section of equity returns (Fama & French, 1993; Koijen et al., 2017). Third is prior work on SDF estimation. Extracting the SDF from asset returns proves difficult in general due to uncertainty regarding the correct parametric form of the SDF. However, non-parametric methods, such as recent work by Ghosh et al. (2016) and Galpin et al. (2017), obviate the
need to specify a functional form for the SDF, and thus avoid model misspecification.

My analysis will employ modern econometric techniques to assess the extent of equity and bond market segmentation in the United States. Specifically, I propose to apply the non-parametric SDF estimation methods of Ghosh et al. (2016) and Galpin et al. (2017), as well as a machine learning-based SDF estimation method motivated by Ghosh et al. (2016), to daily United States equity and bond data. If an SDF estimated from stock and bond returns explains more of the cross-sectional variance in stock and bond returns than does an SDF estimated from stock or bond returns alone, then information in bond yields helps price equities and vice versa. This conclusion would constitute evidence against market segmentation.

My proposed research makes several contributions to the existing literature. First, the non-parametric methods I propose to use can price larger cross-sections of test assets than was previously possible and preclude the need for a benchmark model. Much previous work on jointly pricing equities and bonds has focused on developing structural models of the SDF that provide economic intuition for market integration (Bakshi & Chen, 2005; Campbell et al., 2009; Lettau & Wachter, 2011). Comparatively less work has examined market segmentation from a more direct empirical standpoint (Fama & French, 1993; Koijen et al., 2017). Most of these empirical studies jointly price only a relatively small cross section of assets. Yet it is possible that models that successfully price a small cross section will fail to price a larger set of assets. Moreover, these works usually assess the marginal explanatory power of bond factors beyond a specific benchmark model (e.g. the Fama-French 3 factor model). However, with the “zoo” of factors discovered in the past twenty years, it is possible that newly discovered equity factors subsume the information provided by bond factors (Cochrane, 2011). The methods I propose to use surmount these problems. Second, I propose a novel machine learning-based SDF estimation technique that can potentially overcome some econometric issues faced by existing non-parametric methods. Third, I intend to examine the extent to which equity and bond markets become more segmented during
stressful periods. Although some theoretical work suggests the possibility of time-varying segmentation, previous work has not empirically tested this notion. Fourth, I examine cross-asset-class trading signals based on dual-asset-class SDF estimates (e.g., a signal derived from stock and bond momentum) that have immediate practical applications. Thus, my proposed research provides a thorough assessment of market segmentation between equity and bond markets currently missing from the existing literature.

The remainder of this proposal proceeds as follows. In Section 2 I review the existing literature on market segmentation, joint stock and bond pricing, and SDF estimation. In Section 3 I detail the methods I propose to use and the empirical tests I intend to conduct. In Section 4 I discuss the data I will use. In Section 5 I outline my hypotheses for the tests I will run. Section 6 concludes the proposal.

2 Literature Review

In this section I review the existing literature on market segmentation, joint stock and bond pricing, and SDF estimation.

2.1 Market Segmentation

Market segmentation arises when investors are limited in their ability to share risks in a particular market (Cochrane, 2011). The existing literature highlights market frictions and heterogeneous preferences as the major reasons for limited risk sharing. Previous work on market segmentation mostly focuses two specific cases: segmentation in international equity markets and segmentation in bond markets across the term structure.

2.1.1 International Equity Market Segmentation

Early work on segmentation in international equity markets postulated that market frictions due to structural barriers to the free flow of capital induced “segmentation premia” in na-
tional equity markets where global investors could not effectively arbitrage price differences away. Researchers often appealed to differences in currency areas, political regimes, trade barriers, and capital controls as drivers of segmentation (Solnik, 1974). Furthermore, empirical work found evidence of “mildly segmentation” in international equity markets. For example, Errunza & Losq (1985) found that securities with restricted access due to the above frictions earned “super risk premia.” Nevertheless, as economic liberalization has removed barriers to the free flow of capital, empirical support for segmented international markets has weakened. As a result, theories of segmented markets have been replaced with equilibrium models of capital flows (Errunza & Miller, 2000; Duffie & Strulovici, 2012).

2.1.2 Segmentation Across the Term Structure

Most market segmentation research has focused on segmentation across the term structure. The earliest work in this vein dates back to Modigliani and Sutch’s proposal of the preferred habitat model in the 1960’s (Modigliani & Sutch, 1966, 1967). The preferred habitat model represents one of the many attempts to extend the expectations hypothesis to account for the empirical fact that the yield curve slopes upward (Gürkaynak & Wright, 2012). Under the expectations hypothesis, long run interest rates reflect future expectations of short term interest rates, so risk neutral investors should prove indifferent as to what maturity they lend at. The preferred habitat model, however, holds that many investors are risk averse and often have obligations at fixed maturities in the future (e.g. pension funds). These investors seek certainty in their ability to finance these future obligations, and hence have definite preferences over what maturities to invest at (Modigliani & Sutch, 1967). As a result, “the interest rate is determined by the supply and demand of bonds of that particular maturity” (Gürkaynak & Wright, 2012). That is, bond market segmentation arises along the yield curve due to heterogeneous preferences.

The preferred habitat model has not received much serious attention until recently due to its inability to explain why arbitrageurs don’t enter the market and flatten the yield
curve. 

Modigliani & Sutch (1967) only address this issue on the surface by appealing to transaction costs as an obstacle to such arbitrage. However, interest in the preferred habitat model has reemerged after the 2008 financial crisis due to clear empirical evidence of market segmentation that emerged in that period. Greenwood & Vayanos (2008) and Vayanos & Vila (2009) have developed a preferred habitat model in which arbitrageurs are themselves risk-averse. Others such as Fontaine & Garcia (2011) have focused on the frictions fixed income arbitrageurs face, such as liquidity constraints. Gürkaynak & Wright (2012) hypothesize that in times when transactions costs or risk aversion spike, arbitrageurs may prove insufficiently able to integrate different portions of the yield curve. For example, in late 2008, the U.S. Treasury yield curve exhibited clear segmentation, with short term notes yielding less than longer term bonds with the same maturity dates.

Thus, market segmentation can manifest even in in liquid cross-sections. No obvious frictions inhibit fixed income investors from also participating in equity markets, so one would not expect equity and bond markets to be segmented. As discussed in the Section 2.2, previous work suggests that equity and bond markets are quite integrated. Yet it is conceivable that heterogeneous investor preferences over these two asset classes paired with spikes in arbitrageur risk aversion and other frictions might induce segmentation in extreme periods, such as financial crises.

2.2 The Cross-Section of Bonds and Equities

As discussed above, under the no-arbitrage principle there exists a single SDF that prices all assets, including stocks and bonds. As a result, previous theoretical and empirical work has sought to unify equity and bond pricing.

2.2.1 Theoretical and Empirical Links between Bonds and the SDF

In this particular application, factors derived from bond yields should help price the cross-section of equity returns for economically intuitive reasons. Under a time-separable utility
function, the SDF represents the growth rate of an investor’s marginal utility (Campbell, 2000). By construction, an asset’s expected excess return is proportional to the negative of it’s covariance with the SDF. Since stocks are negatively correlated with the SDF, they perform poorly when the SDF is high, or equivalently, when marginal utility is high. As a result, equities command a risk premium because they often perform poorly in times of high marginal utility for investors, such as recessions or other negative economic events (Cochrane, 2017). Thus, to the extent that factors derived from bond yields can predict future economic activity, they should inform the SDF.

A large body of empirical work finds that bond factors do forecast future economic activity. Macroeconomic shocks impact investor expectations about economic fundamentals (e.g. inflation, growth) and expectations about policy responses to these fundamentals, and thus contemporaneously affect the term structure (Campbell et al., 2009). At the same time, bond factors also predict future economic activity. Harvey (1988) establishes that the real term structure contains predictive information of consumption growth. Stock & Watson (1989) find that long-term/short-term Treasury yield spreads serve as useful leading indicators of a variety of macroeconomic variables. Many authors link term structure information to future GDP growth (Estrella & Hardouvelis, 1991; Plosser & Rouwenhorst, 1994; Haubrich & Dombrsky, 1996). In more recent work, Brooks (2011) finds that “tent factor” of Cochrane & Piazzesi (2005), a linear combination of forward rates, forecasts unemployment at quarterly frequencies. Gilchrist & Zakrajšek (2012) find that corporate bond credit spreads forecast payroll employment, unemployment, and industrial production at horizons of up to a year. Kojien et al. (2017) demonstrate that many bond factors (e.g. the Cochrane-Piazzesi tent factor, the slope factor of Litterman & Scheinkman (1991), and others) forecast future economic activity, measured by the Chicago Fed National Activity Index and GDP, at business cycle horizons of two to three years.
2.2.2 Cross-Sectional Bond and Stock Pricing

These theoretical and empirical results have motivated work on cross-sectional bond and stock pricing. This body of research provides evidence that bond factors do help explain equity returns and vice versa. I divide this work into two categories. The first is empirical investigations that conduct regressions to estimate the empirical prices of risk of various factors. Fama & French (1989) find that dividend yields forecast bond returns, while the “default-premium,” the credit spread between AAA bonds and the market portfolio of corporate bonds, and the “term-premium,” the spread between AAA yields and one-month Treasury yields, forecast excess equity returns. Fama & French (1993) demonstrate that the default and term premia factors add significant explanatory power to the cross-section of equity returns beyond the market, SMB, and HML factors. They also find that these equity factors help explain the cross-section of low-grade corporate bond returns. Cochrane & Piazzesi (2005) show that their tent factor predicts not only excess returns of one to five year Treasuries, but also excess stock returns at one-year horizons. Koijen et al. (2017) use bond factors to empirically explain the value premium. They demonstrate that the cash flows of value stocks have greater exposure than those of growth stocks to bond factors that predict economic activity at business cycle horizons. As a result, value stocks command a premium because their cash flows are more significantly impacted in times when investor marginal utility is already high. Thus, this body of work finds that empirically bond factors do help explain the equity returns.

The second category of work in cross-sectional stock and bond pricing focuses on structural models of equity and bond returns. These authors propose models, usually affine term structure models, of the SDF that they calibrate to observed data. They then compare the moments yielded by return series generated under the model to observed moments to assess model’s realism. Compared to the purely empirical investigations discussed above, this vein of literature places more emphasis on theoretical justification for the model structure and input variables chosen. Bakshi & Chen (2005) propose a stock valuation model based on
a single factor term structure model of the SDF and obtain relatively small pricing errors. 
Wachter (2006) proposes a consumption based model that produces realistic bond and stock 
volatility and a high equity premium. Bekärt et al. (2009) propose an affine term structure 
model of the SDF based on fundamental macroeconomic variables such as inflation and con-
sumption growth. They find that this model matches the empirical volatilities of dividend 
and consumption growth, and generates a large equity premium and low risk-free rate. Let-
tau & Wachter (2011) achieve similar results with a model parametrized to dividend growth, 
inflation, and short-term real rates, and in particular have some success in matching the 
value premium. Departing from the macroeconomic basis of most of these models, Gabaix 
(2012) finds that a structural model of the SDF based on rare disasters explains a variety 
of asset pricing puzzles, including jointly pricing stocks and bonds. Campbell et al. (2009) 
employ a multifactor term structure model based on real interest rates, inflation, and a state 
variable to explain the time-varying covariance of stock and bond returns.

The cross-sectional stock and bond pricing literature suggests deep theoretical under-
pinnings for why bond factors should help price equities and vice versa. Empirical results 
corroborate this theory. As discussed in Section 2.3, modern econometric techniques have 
the ability to better quantify the extent of this cross-asset-class pricing ability.

2.3 Empirical SDF Estimation

The SDF is a fundamentally important object in asset pricing. Indeed, if one could perfectly 
characterize the SDF and the stochastic payoff process of a given asset, he or she could 
perfectly price that asset. As a result, many researchers have sought methods to estimate the 
SDF. The ideal SDF model would directly link asset returns to underlying macroeconomic 
variables, describing the observed data well and providing theoretical insight (Campbell, 
2000). Unfortunately, structurally motivated SDF models, such as those discussed in Section 
2.2, can usually at best only qualitatively match certain aspects of the observed data. For 
example, many structural models do yield a sufficiently volatile SDF as determined by the
Hansen-Jagannathan lower bound (Hansen & Jagannathan, 1991). These models cannot, however, generate return patterns that fit observed asset returns well (Cochrane, 2017). As a result, many researchers have developed methods to extract the SDF directly from observed asset prices. Even if these methods cannot necessarily illuminate the underlying macroeconomic fundamentals of the SDF, they often prove practically useful (e.g. for risk management, derivatives pricing, and asset allocation). Moreover, these methods can help identify which systematic risk factors empirically impact asset returns.

2.3.1 SDF Estimation from Options Data

Most previous work on extracting the SDF directly from asset prices has used options data. Options prove theoretically appealing for this application because they contractually specify payoffs in different states of nature and trade on exchanges. Early work in this area designed parametric techniques to extract the state price density (closely related to SDF) from option prices (Ross, 1976; Banz & Miller, 1978; Breeden & Litzenberger, 1978). These approaches impose strict assumptions on the distribution of the SDF, and hence fall victim to the dangers of model misspecification (Hansen, 2014).

Thus, subsequent work focused on developing non-parametric methods that more accurately describe observed data due to their lack of unrealistic assumptions. For example, Jackwerth & Rubinstein (1996) extract the risk-neutral probability distribution, which is closely related to the SDF, from daily S&P 500 index options data. Aït-Sahalia & Lo (1998) and Aït-Sahalia & Lo (2000) develop non-parametric methods to extract the state price density, also closely related to the SDF, from options prices. Rosenberg & Engle (2002) present a non-parametric method to estimate the SDF using S&P 500 index options data and a stochastic volatility model of the S&P 500. The limitation of these models is their reliance on options data. Options are not nearly as liquid as stocks. Indeed at strike prices far from the spot price, options can be quite illiquid. Thus, data quality can significantly limit the performance of these methods. To cope with this issue, these papers use S&P 500
index options, which are perhaps the most liquid options. As a result, one cannot easily apply these methods to other stocks with less liquid options chains, and certainly not to other asset classes such as bonds, which do not have exchange traded options.

2.3.2 General Non-Parametric SDF Estimation

Fortunately, recent work has presented methods to extract the SDF from general asset return data, thereby allowing researchers to estimate the SDF directly from, for example, equity return data. Ghosh et al. (2016) provide an information-theoretic approach to estimating the SDF. Given a cross-section of assets (the authors focus on equity portfolios), the proposed method minimizes a particular entropy-based loss function (the Kullback-Leibler Information Criterion), subject to the constraint that the SDF pricing equation holds. The resulting optimization problem has a solution that expresses the SDF as a nonlinear function of the Lagrange multipliers of the optimization problem and the observed asset returns. The authors construct a rolling out-of-sample estimate of the SDF by computing the Lagrange multipliers from previous data, and then calculating the SDF over the evaluation period from those out-of-sample estimates and the contemporaneous cross-sectional returns. They then use this “Information SDF” (I-SDF) and its linear projection onto the return space, called the “Information Portfolio” (I-P), as single factor pricing models. The I-SDF and I-P series allow one examine how well the SDF extracted from a given set of portfolios prices those portfolios out of sample. Ghosh et al. (2016) find that both the I-SDF and I-P cross-sectionally price the test assets better than Fama-French three-factor and Carhart four-factor models. The I-SDF provides a flexible and computationally tractable way to extract the SDF from any set of test assets. Thus, one can easily extend this analysis to other classes by, for example, including bonds in the cross-section of test assets.

Galpin et al. (2017) conduct a similar but slightly different analysis. These authors use the I-SDF of Ghosh et al. (2016), which they call the exponential tilting estimator, and another non-parametric SDF estimator motivated by the continuously-updated estimator of Hansen
et al. (1996) to estimate the SDF directly from a set of systematic risk factors, not from a set of test assets. In this way, Galpin et al. (2017) obtain the Lagrange multipliers from the pricing constraints for the set of test factors as in Ghosh et al. (2016). They then test the significance of these Lagrange multipliers in the SDF via tests developed by Newey & Smith (2004). These tests allow the authors to determine which factors add significant information to the SDF when conditioned on the other test factors. The authors argue that adding in auxiliary test assets “muddles inference,” whereas their method can directly determine which factors prove economically significant. They find that four factors suffice to characterize the SDF: the market, profitability, investment, and value-profitability-momentum. This method also proves easily extensible to other asset classes.

3 Methodology

In this section I detail the methods I propose to use and the empirical tests I intend to conduct. To ensure robustness, I will test for equity and bond market segmentation in three different ways.

3.1 Testing for Segmentation via SDF Estimation

I first detail two non-parametric SDF estimation techniques and then outline the statistical tests I will use to test for market segmentation.

3.1.1 SDF Estimation Method of Ghosh et al. (2016)

Ghosh et al. (2016) use an information-theoretic approach to non-parametrically extract the SDF from a cross section of test assets. Specifically, they derive the “least-informative” SDF that prices all of the assets in the sample period. Least-informative in this context means the SDF corresponding to the risk-neutral probability measure that deviates least from the hypothetical physical probability measure, while still pricing all assets. Precisely,
Ghosh et al. (2016) present the following non-parametric, maximum-likelihood estimate of the risk-neutral probability measure $\mathbb{Q}$ given the physical probability measure $\mathbb{P}$:

$$\argmin_{\mathbb{Q}} D(\mathbb{Q}\|\mathbb{P}) \text{ s.t. } \mathbb{E}_{\mathbb{Q}}[\mathbf{R}_t^e] = 0 \in \mathbb{R}^n, \quad (3.1)$$

where $\mathbf{R}_t^e \in \mathbb{R}^n$ is the vector of excess returns of our $n$ test assets in period $t$, and $D(\mathbb{Q}\|\mathbb{P})$ is the Kullback-Leibler Information Criterion (KLIC). The KLIC is a measure of the extent to which one probability distribution deviates from another. Thus, $\mathbb{Q}$ as given by $(3.1)$ is the risk-neutral measure that requires the least additional information over the physical probability measure but that still prices all test assets. Some measure-theoretic simplifications to $(3.1)$ yields the following equivalent optimization problem:

$$\argmin_{\{M_t\}_{t=1}^T} \frac{1}{T} \sum_{t=1}^T M_t \ln(M_t) \text{ s.t. } \frac{1}{T} \sum_{t=1}^T M_t \mathbf{R}_t^e = 0, \quad (3.2)$$

where $M_t$ is the value of the SDF at time $t$. As Ghosh et al. (2016) note, $(3.2)$ has the following solution:

$$M_{t}^{info} = \frac{e^{\lambda_T^\top \mathbf{R}_t^e}}{\sum_{t=1}^T e^{\lambda_T^\top \mathbf{R}_t^e}}, \quad \lambda_T = \underset{\theta}{\arg\min} \frac{1}{T} \sum_{t=1}^T e^{\theta^\top \mathbf{R}_t^e}, \quad (3.3)$$

where $\lambda_T \in \mathbb{R}^n$ is the vector of Lagrange multipliers required for the pricing constraint in $(3.2)$ to hold. The notation $M_{t}^{info}$ follows from Ghosh et al. (2016) naming their estimator the “Information SDF.” To summarize, $M_{t}^{info}$ as given by $(3.3)$ corresponds to essentially the least-complex risk-neutral probability measure that still prices all assets. Note that $(3.3)$ gives a way to estimate the SDF out-of-sample: extract $\lambda_T$ from a previous sub-sample $\{t_0 - s, t_0 - s + 1, \ldots, T - s\}$, $s > T - t_0$, and calculate each $M_t$ based on $\mathbf{R}_t^e \forall t \in \{t_0, \ldots, T\}$.

In the next section, I propose a similar non-parametric SDF method.
3.1.2 Neural Network-Based SDF Estimation

The non-parametric SDF estimation method of Ghosh et al. (2016) motivates a neural network-based SDF estimation procedure. Neural networks, also known as “deep nets,” are a machine learning method that uses a set of training examples to non-parametrically learn a continuous function between, in this setting, a vector-valued input variable and a vector-valued output variable. Note that (3.3) specifies the SDF $M_t$ as a non-linear function of $R_t^e$ and the parameter vector $\lambda_T$ obtained from the pricing constraint. In the same way, a neural network learns a parameter vector $\omega$ that yields the “best” mapping between the input and output variables according to some loss function.

Precisely, let $\sigma(x) = 1/(1+e^{-x})$ be the logistic function. Note that $\sigma$ maps $x$ continuously into the range $(0, 1)$, exactly the domain of the SDF. As above, let $R_t^e \in \mathbb{R}^n$ be the vector of excess asset returns in period $t$. Motivated by the pricing constraint in (3.2), let

$$L(\omega; R_t^e) = \frac{1}{n} \| R_t^e \sigma(\omega^T R_t^e) \|_2^2 = \frac{1}{n} \sum_{i=1}^{n} (R_{t,i}^e \sigma(\omega^T R_t^e))^2,$$

be the least-squares loss function, where $R_{t,i}^e$ is the excess return of the $i$-th asset in period $t$. Note that if we let $M_t = \sigma(\omega^T R_t^e)$, then minimizing $L(\omega; R_t^e)$ corresponds to finding an SDF that forces the pricing constraint to (almost) hold in period $t$. Thus, let

$$M_t^{deep} = \sigma(\omega^T R_t^e), \quad \omega = \arg\min_{\theta} \frac{1}{T} \sum_{t=1}^{T} L(\theta; R_t^e),$$

so $M_t^{deep}$ is the SDF that forces the pricing constraint to hold in the full sample. (3.5) is typically solved in the deep learning literature via stochastic gradient descent. For brevity, I omit the details of stochastic gradient descent and refer the reader to Heaton et al. (2016), a review of the uses of deep learning in finance, for more details. Note that $M_t^{deep}$ can be estimated in a rolling out-of-sample fashion in exactly the same fashion described above for $M_t^{info}$. Figure 3.1.2 illustrates the neural network described in this section graphically.
This SDF estimator bears several structural similarities to that of Ghosh et al. (2016). Both estimators model the SDF as a nonlinear function based on a parameter vector learned from the pricing constraint. Indeed, the specific nonlinear mappings in both cases are functionally similar. The difference between the two methods is the manner in which the parameter vectors $\lambda_T$ and $\omega$ are learned. Yet comparing the equation for $\lambda_T$ in (3.2) to the equation for $\omega$ in (3.5), we see that the only difference in the ways $\lambda_T$ and $\omega$ are learned is the particular loss function chosen. Thus, this neural network SDF estimation technique is no more complex than the estimation method of Ghosh et al. (2016). However, the neural network approach offers several potential advantages in this setting. First, whereas Ghosh et al. (2016) acknowledge that their method requires a large time series dimension relative to the cross-sectional dimension, neural networks have proven useful in many high-dimensional applications (e.g. image recognition), and thus might offer superior performance in this setting as we expand the cross-section of assets. Second, since I seek to model the SDF in an out-of-sample fashion, overfitting of the parameter vector to a previous subsample represents a serious concern. Deep learning offers several methods to prevent overfitting (e.g. regularization, dropout layers) that manifest as slight modifications to the optimization problem in (3.5). Thus, this neural network estimator may provide a more informative SDF than that yielded by the method of Ghosh et al. (2016). At the very least, this method will provide an additional robustness check for the results yielded by the other tests I use.
3.1.3 Tests of Market Segmentation

I propose to estimate the SDF via (3.3) and (3.5) in a rolling out-of-sample fashion from three sets of assets: a set of equity test assets ($S_{Equity}$), a set of bond test assets ($S_{Bond}$), and the union of these two sets ($S_{All}$). Denote these six SDFs as $M^{Equity,j}$, $M^{Bond,j}$, and $M^{Dual,j}$, $j \in \{info, deep\}$, respectively. Now in the manner of Ghosh et al. (2016), $\forall i \in \{Equity, Bond, Dual\}, j \in \{info, deep\}$ construct tradeable factor-mimicking portfolios by linearly projecting each SDF onto the return space:

$$w^{i,j} = -\frac{\hat{b}}{|\hat{b}^T\eta|}, \quad (\hat{a}, \hat{b}) = \arg\min_{a, b} \frac{1}{T} \sum_{t=1}^{T} (M^{i,j}_t - a - b^T R^e_t)^2,$$

(3.6)

where $w^{i,j}$ is the vector of portfolio weights and $\eta$ is a vector of conformable ones. Let $R^{e,i}_t, i \in \{Equity, Bond, Dual\}$ be the vector of excess returns for each set of test assets. Then $R^{e,i,j}_t = w^{i,j^T} R^{e,i}_t \in \{Equity, Bond, Dual\}, j \in \{info, deep\}$ is the factor-mimicking portfolio excess return for period $t$, where the weight vector $w^{i,j^T}$ has been calculated out-of-sample from a previous subsample. Thus, $R^{Equity,j}_t, R^{Bond,j}_t, R^{Dual,j}_t, j \in \{info, deep\}$, represent six pricing factors.
I will use these six pricing factors to test for market segmentation in two ways.

**Benchmark-free tests of market segmentation**

My first test of market segmentation involves two benchmark-free cross-sectional regressions. By obviating the need to specify a benchmark model, I strip out the potential statistical biases that arise from using the wrong benchmark model (e.g. omitted variable bias). The first pass of this test is to obtain factor loadings for each asset on its corresponding single-asset-class SDF and on the dual-asset-class SDF. For \( j \in \{info, deep\}, i \in \{Equity, Bond\} \), regress the excess returns for asset \( k \in S_i \) on \( R^{e,i,j}_t \) and \( R^{e,Dual,j}_t \):

\[
R^{e,i,j}_t = \alpha + \beta_k R^{e,i,j}_t + \epsilon_t,
\]
\[
R^{e,Dual,j}_t = \alpha + \beta_k R^{e,Dual,j}_t + \epsilon_t.
\]

Now cross-sectionally regress the sample expected returns for each assets \( k \in S_i \), denoted \( \overline{R}^{e,i}_k \), on the vectors of factor loadings:

\[
\overline{R}^{e,i}_k = \alpha + \theta^{i} \beta_k^{i,j} + \epsilon_k, \tag{3.7}
\]
\[
\overline{R}^{e,i}_k = \alpha + \theta^{Dual} \beta_k^{Dual,j} + \epsilon_k. \tag{3.8}
\]

Here, \( \theta^{i} \) and \( \theta^{Dual} \) represent the risk premia earned by exposure to the factor-mimicking portfolios for \( M^{i,j} \) and \( M^{Dual,j} \). If the full model (3.8) has significantly better fit than the nested model (3.7) (as determined by an F-test, a likelihood-ratio test, etc.), then one would conclude that the dual-asset-class SDF explains significantly more of the variance in cross-sectional stock or bond returns than does a single-asset-class SDF. This conclusion would provide evidence against market segmentation. I will conduct this test both in the full sample and in stressful subsamples.

**Comparison to equity and bond benchmarks**

My second test of market segmentation will compare the average alphas for all test assets in each asset class under a benchmark model and under that same benchmark model
augmented with the single and dual-asset-class SDF-mimicking portfolios. I will use the Fama-French five factor model as the equity benchmark (Fama & French, 2016), and the principal-component extracted level-slope-curvature three factor model as the bond benchmark (Litterman & Scheinkman, 1991).

For each asset \( k \in S_{\text{Equity}} \), conduct the following three time series regressions for all \( j \in \{info, deep\} \):

\[
R_{t,k}^e = \alpha_k + b_k R_{M,t}^e + s_k \text{SMB}_t + h_k \text{HML}_t + r_k \text{RMW}_t + c_k \text{CMA}_t + \epsilon_{k,t}, \tag{3.9}
\]

\[
R_{t,k}^e = \alpha_k^{j_0} + b_k^{j_0} R_{M,t}^e + s_k^{j_0} \text{SMB}_t + h_k^{j_0} \text{HML}_t + r_k^{j_0} \text{RMW}_t + c_k^{j_0} \text{CMA}_t + \beta_k^{j_0} R_{k}^{\text{Equity},j} + \epsilon_{k,t}, \tag{3.10}
\]

\[
R_{t,k}^e = \alpha_k^{j_1} + b_k^{j_1} R_{M,t}^e + s_k^{j_1} \text{SMB}_t + h_k^{j_1} \text{SMB}_t + r_k^{j_1} \text{RMW}_t + c_k^{j_1} \text{CMA}_t + \beta_k^{j_1} R_{k}^{\text{Dual},j} + \epsilon_{k,t}. \tag{3.11}
\]

Here \( R_{M,t}^e \) is the excess return of the market, \( \text{SMB}_t \) is the size factor, \( \text{HML}_t \) is the value factor, \( \text{RMW}_t \) is the profitability factor, and \( \text{CMA}_t \) is the investment factor, all as described in Fama & French (2016).

For each asset \( k \in S_{\text{Bond}} \), conduct the following three time series regressions for all \( j \in \{info, deep\} \):

\[
R_{t,k}^e = \alpha_k + l_k \text{Level}_t + s_k \text{Slope}_t + c_k \text{Curvature}_t + \epsilon_{k,t}, \tag{3.12}
\]

\[
R_{t,k}^e = \alpha_k^{j_0} + l_k^{j_0} \text{Level}_t + s_k^{j_0} \text{Slope}_t + c_k^{j_0} \text{Curvature}_t + \beta_k^{j_0} R_{k}^{\text{Bond},j} + \epsilon_{k,t}, \tag{3.13}
\]

\[
R_{t,k}^e = \alpha_k^{j_1} + l_k^{j_1} \text{Level}_t + s_k^{j_1} \text{Slope}_t + c_k^{j_1} \text{Curvature}_t + \beta_k^{j_1} R_{k}^{\text{Dual},j} + \epsilon_{k,t}. \tag{3.14}
\]

Here \( \text{Level}_t, \text{Slope}_t, \) and \( \text{Curvature}_t \) are the first, second, and third principal components extracted from the time series of 1, 3, 5, 7, and 10 year zero-coupon Treasury yields, as described in Litterman & Scheinkman (1991).
For \( j \in \{info, deep\}, i \in \{Equity, Bond\} \), let

\[
\overline{\alpha}_i = \frac{1}{|S_i|} \sum_{k \in S_i} \alpha_k, \quad \overline{\alpha}_i^{j,0} = \frac{1}{|S_i|} \sum_{k \in S_i} \alpha_k^{j,0}, \quad \overline{\alpha}_i^{j,1} = \frac{1}{|S_i|} \sum_{k \in S_i} \alpha_k^{j,1}.
\] (3.15)

If \( \overline{\alpha}_i^{j,0} \) is significantly less than \( \overline{\alpha}_i \), then the single-asset-class SDF adds significant additional pricing power over the benchmark model. If \( \overline{\alpha}_i^{j,1} \) is significantly less than \( \overline{\alpha}_i^{j,0} \), then the dual-asset-class SDF adds significant additional pricing power over the single-asset-class SDF.

For example, if \( \overline{\alpha}_i^{j,1}_{Equity} \ll \overline{\alpha}_i^{j,0}_{Equity} \), then bond factors help price equities, which is evidence against market segmentation.

The benefit of this test is that, even though it requires specifying benchmark models, it can impart economic intuition about the extent of market integration in the form of the basis point reduction in average alpha due to each of the SDFs.

### 3.2 Testing for Segmentation via SDF Information

In this section, I detail the method of Galpin et al. (2017), which tests which assets add significant information to the SDF. Galpin et al. (2017) establish that in this setting,

\[
\lambda_T \xrightarrow{D} \mathcal{N}(0, \mathbf{P}), \quad \mathbf{P} = \frac{1}{T} \sum_{t=1}^{T} M_t^{info} \mathbf{R}_t^e \mathbf{R}_t^{e\top} \in \mathbb{R}^{n \times n},
\] (3.16)

where \( n \) is the number of test assets and \( \lambda_T \) is the vector of Lagrange multipliers used in the SDF estimation method of Ghosh et al. (2016) defined in (3.3). Thus, a test of \( H_0 : \lambda_{T,k} = 0 \) is a test of if the pricing constraint for asset \( k \) binds, and so is a test of if asset \( k \) adds a significant amount of information to the SDF.

Let \( \lambda_T^j \) represent the \( \lambda_T \) vector derived from the set of test assets \( S_i, i \in \{Equity, Bond, All\} \).

I propose to test, in the full sample and in stressful subsamples, the following joint null hy-
hypothesis for $i \in \{Equity, Bond\}$:

$$H_0^i : \lambda_{T,k}^{All} = 0, \forall k \in S_i.$$  

Rejecting $H_0^{Equity}$ would imply that the equity test assets collectively add a significant amount of information to the SDF. Similarly, rejecting $H_0^{Bond}$ would imply that the bond factors collectively add a significant amount of information to the SDF. Moreover, by comparing the significance of elements in $\lambda_T^{Equity}$ and $\lambda_T^{Bond}$ to the significance of the corresponding entries in $\lambda_T^{All}$, we can determine which information provided by equity assets is subsumed by bond assets and vice versa. Additionally, estimating $\lambda_T^{All}$ across subsamples and plotting its elements over time will illustrate any time-variation in the information content of each test asset. Thus, this method provides another benchmark-free test of market segmentation.

### 3.3 Dual-Asset-Class Trading Signals

The third way I will test for market segmentation is by examining the performance of dual-asset-class trading strategies. I propose to compare the performance of common equity strategies and variants of those strategies informed by a dual-asset-class SDF. If the dual-asset-class SDF significantly improves performance, then bonds help price equities. This conclusion would suggest market integration.

I propose to examine trading signals for the value and momentum anomalies based on sorted equity and bond portfolios. I will use the standard Fama-French decile sorts as the equity portfolios. Following the setup of Brooks & Moskowitz (2017), I will use the 1, 3, 5, 7, and 10 year U.S. Treasury zero coupon bonds sorted on real yields (current yield minus maturity-matched expected inflation) as the bond value assets, and these same bonds sorted on trailing 12-month returns as the momentum assets. Consider the following three strategies based on these assets:

1. **Control strategy**: Long the first equity decile and short the tenth decile.
2. **Single-SDF strategy**: Extract the SDF out-of-sample from the ten equity portfolios by (3.3) and (3.5). Project each SDF onto the return space of these portfolios via (3.6) to create two tradeable anomaly portfolios. As noted by Ghosh et al. (2016), these portfolios should represent the tangency portfolios of these ten assets.

3. **Dual-SDF strategy**: Extract the SDF out-of-sample from the ten equity portfolios and the five bonds by (3.3) and (3.5). Project each SDF onto the return space of these equity portfolios via (3.6) to create two tradeable anomaly portfolios.

If the single-SDF strategy performs significantly better than the control strategy (on the basis of Sharpe ratio, Fama-French five factor alpha, etc.), then the SDF we estimate does uncover a better asset allocation, and perhaps identifies the tangency portfolio. If the dual-SDF strategy performs significantly better than the single-SDF strategy, then information in bond yields helps price and trade equities, which would be evidence against market segmentation. In addition to providing another robustness check to my other tests of market segmentation, testing these dual-asset-class trading signals also highlights the practical value of the SDF estimation techniques I use.

4 **Data**

In this section I discuss the data I will use in this research. I propose to use daily U.S. stock and bond returns. To ensure robustness, I will use a wide cross section of test equity portfolios. Specifically, I intend to use daily decile sorts on size, value, momentum, investment, and profitability, obtained from Kenneth French’s data library. This data set covers the period from July 1963 to September 2017 (13,600 observations). In order to capture information from the full yield curve, I will use U.S. Treasury zero-coupon bonds with maturities of 1, 3, 5, 7, and 10 years, the yield curve slope between the 10 and 2 year yields, the yield curve "butterfly" of 5–Average(2, 10) year yields, and the tent factor of Cochrane & Piazzesi (2005) (a linear combination of 1-5 year Treasury forward rates, which I will bootstrap from the
1, 3, and 5 year yields). The CRSP Fixed Term Indexes dataset has daily data on these maturities from June 1941 to December 2016.

I will use the NBER recession periods as the stressful periods for my subsample analyses. For the expected inflation rates used to form the bond value assets, I will use quarterly 1 and 10 year CPI inflation forecasts obtained from the Federal Reserve Bank of Philadelphia. This dataset covers the period from 1991 to 2017. I will take weighted geometric averages of the 1 and 10 year forecasts to obtain the inflation forecasts for intermediate lengths of time.

5 Hypotheses

In this section, I summarize the empirical tests I propose to conduct and outline my priors on the results of these tests. Overall, based on the success of previous joint bond and stock pricing efforts, as discussed in Section 2.2, I expect to find evidence of market integration. No-arbitrage is a fairly weak assumption, especially in liquid cross sections, so theory suggests a single SDF will price stocks and bonds. Furthermore, previous work, such as Fama & French (1993), has found bond factors to be informative in the cross-section of equity returns and vice versa. Even though, as discussed in section 2.1.2, the preferred habitat hypothesis and work on term structure segmentation suggest that equity and bond markets may display signs of segmentation in stressful periods, I maintain the same prior here due to the results of previous full-sample analyses. Table 1 exhibits the empirical tests I propose to run and my priors on the results of each of these tests.
<table>
<thead>
<tr>
<th>Test</th>
<th>Statistical $H_0$</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-test/likelihood ratio test of single vs. dual-asset-class SDF in cross-sectional regression</td>
<td>$\forall j \in {\text{info}, \text{deep}}$</td>
<td>$\forall j \in {\text{info}, \text{deep}}$, $H_0^j$ will be rejected</td>
</tr>
<tr>
<td>Comparison of average alphas under benchmark and SDF-augmented benchmarks</td>
<td>$\forall i \in {\text{Equity, Bond}}$, $j \in {\text{info}, \text{deep}}$</td>
<td>$\forall i \in {\text{Equity, Bond}}$, $j \in {\text{info}, \text{deep}}$, $H_0^{1,i,j}$ and $H_0^{2,i,j}$ will be rejected.</td>
</tr>
<tr>
<td>Significance tests of equity and bond Lagrange multipliers in SDF estimate</td>
<td>$\forall i \in {\text{Equity, Bond}}$, $H_i^1: \lambda^\text{All}_i = 0, \forall k \in S_i$</td>
<td>$\forall i \in {\text{Equity, Bond}}$, $H_0^i$ will be rejected</td>
</tr>
<tr>
<td>Comparison of control, single-SDF, and dual-SDF trading strategies</td>
<td>$\forall a \in {\text{value, momentum}}$, $j \in {\text{info, deep}}$</td>
<td>$\forall a \in {\text{value, momentum}}$, $j \in {\text{info, deep}}$, $H_0^{a,j}$ will be rejected</td>
</tr>
</tbody>
</table>

### 6 Conclusion

I seek to rigorously test the extent of market segmentation between equity and bond markets. Previous work suggests equity and bond markets should prove integrated, with the possible exception of some segmentation arising during stressful periods. I propose to employ new econometric techniques that non-parametrically estimate the SDF in order to avoid model misspecification in both the SDF estimation stage and in the asset pricing stage. In addition to applying recent SDF estimation techniques from the literature, I propose a new machine learning-based method. Furthermore, I propose to conduct comparisons of trading strategies based on these SDF estimators to not only provide another test of segmentation, but also illustrate the practical value of these methods.
References


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